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THE GEOMETRY OF COHESIVE MAGNET-COIL WINDING

Summary - During the winding of a magnet-coil, the wire is pulled tightly across the surface of the inner part that has already been wound. If the wire is not a geodesic in the surface, it will tend to slip laterally. Classical differential geometry is applied to the problem of winding the coil so as to minimize this tendency, subject to the constraint that a prescribed magnetic field be produced.

NOTATION

D	subset of euclidean space
x	point (x_1, x_2, x_3) in D
r	curve in D
s	arc-length parameterization of r
$T(x)$	tangent to r at x
$N(x)$	principal normal to r at x
$B(x)$	binormal to r at x
$\tau(x)$	torsion of r at x
κ	curvature of r
$\delta(x)$	Darboux vector of r at x
$J(x)$	current density (vector) at x
L	layering function (scalar) on D

L_0	optimal layering function on D
W	windability of a current distribution
w	windability in a given layering
R	equivalence relation on D
D/R	quotient manifold of D by R
L/R	quotient of L by R
Q	quotient mapping from D to D/R
$*$	adjoint (of a linear transformation)
e_1, e_2	real-valued functions of x
g	linear differential form on D/R
g_0	optimal linear differential form on D/R
G	linear differential form on D
u_1, u_2	local coordinates in D/R
f_1, f_2	real-valued functions of u_1 and u_2
Θ	angle
α	angle
a	inner radius of annulus
b	outer radius of annulus
h	height of annulus
c	constant
S	strip of geodesics containing r
λ	parameterization of strips parallel to $S_{r,0}$
$S_{r\lambda}$	strips parallel to $S_{r,0}$
ε	half-width of strip $S_{r,0}$
C	linear space of current distributions
M	linear space of magnetic fields
$\lvert v \rvert$	length of vector v

$\cdot$ inner product

∇ $\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$

INTRODUCTION

The oldest branch of differential geometry is its theory of curves and surfaces in euclidean space. It deals with concrete objects immediately accessible to our naive spatial intuition. It is a beautiful theory to contemplate, but it was also meant by its creators to be applied to real problems. The purpose of this paper is to apply it to the mechanics of magnet-coil winding.

A curve in a surface is a *geodesic* if and only if it connects any two of its points (not too far apart) by an arc that is shorter than any other arc in the surface connecting those two points. If the surface is a plane, then its geodesics are straight lines. If the surface is a sphere, the geodesics are great circles. In general (Brand [1]), "If a weightless flexible cord is stretched over a smooth surface between two of its points, its tension is constant and the line of contact is a geodesic". If one tries to wind a "weightless flexible cord" along a curve that is not geodesic, the cord will tend to slip to become a geodesic. In the process, it may slip entirely off of the supporting surface.

This obvious fact will be enlarged into a mathematical model of a magnet coil that, though highly idealized, does predict the lateral forces on the wire during coil fabrication that arise from these purely geometric considerations. A coil is wound in layers with the wire in each layer pulled tightly against the layer beneath it. A layer is idealized as a smooth surface and a wire as a curve in the surface. Then the extent that it deviates from a geodesic, and the associated tendency towards lateral slippage, can be computed by classical formulae. In this idealization the wire has no thickness so the model does not include the support it gets from the one of its two neighbors that has already been wound in the layer. However, layers are usually wound alternately from one side to the other so the support on one side that may have been crucial in the prevention of slippage in one layer will be absent in the next, and the lateral forces predicted by the theory should still be taken into account. In any case, the slippage tendency of the layer as a whole is modeled.

Not modeled at all are the forces that the magnetic field will produce later when the coil is energized. There is no field to contend with while the coil is being wound.

A solenoid can be modeled by helices wrapped on concentric cylindrical surfaces. They are geodesics so the kinds of forces we are concerned with are perpendicular to the surfaces and there is no slippage problem. On the other hand there are important coil shapes, for example saddle coils used in magnetohydrodynamic power generation (dipole) and accelerator beam steering (quadrupole), where the slippage problem can be so serious as to cause disassembly of the coil during fabrication. The precipitating circumstance of this paper was the threat of just such an occurrence, called to my attention by R.P.Smith, at the Superconducting Magnet Group in Argonne National Laboratory.

A measure of the problem is the size of the coefficient of friction needed to prevent lateral slippage. The coefficient is the tangent of the "angle of friction." The complement of this angle must be less than the angle between the surface and the osculating plane to the curve at that point. (If as three non-collinear points on a smooth curve approach a limit point the plane they determine approaches a unique limiting plane, that plane is called the osculating plane to the curve at the limit point. The curve is a geodesic in a smooth surface if and only if the osculating plane, at every point on the curve where it exists, is perpendicular to the surface at that point. The force exerted by the wire on the surface is in the direction of the principal normal to the curve, a vector in the osculating plane that is perpendicular to the curve. This can be seen from a parallelogram of forces determined by the three points as they approach their limit.)

So far we have assumed that the layering of the coil was fixed. But a layering is not determined by the current distribution the coil is designed to carry. For example, early in the winding of a solenoid one end of the coil could be disproportionally built up so that the layers were concentric cones. Circles are not geodesic on right circular cones so the wires would tend to slip down towards their vertices.

An *optimal* layering of a current distribution will be defined as one for which the needed coefficient of friction is minimal. The *windability* of a current distribution will be defined as the reciprocal of the smallest coefficient of friction that would keep the wire from slipping laterally in an optimal layering. Thus the windability of a solenoidal current distribution is infinite because in an optimal layering (concentric cylinders) no frictional forces are needed (ideally).

There are other distributions for which no such layering can be found. If the solenoid is skewed by an affine transformation, there is no way the wire can be wound as a geodesic. The windability for skewed solenoids will be computed and shown to be finite. This artificial example is used to illustrate the theory because of its mathematical simplicity. Saddle coils have an inconveniently low windability but the computation is not as simple.

A procedure is described for finding optimal layerings.

Finally, it is shown that any current distribution can be approximated arbitrarily closely by another whose windability is as large as desired. The price paid for the high windability of the approximating distribution may, however, be an infeasibly complicated supporting structure.

DEFINITION OF WINDABILITY

All the mathematics that we will need was already familiar to the nineteenth century geometers. It is almost entirely contained in chapters III and VIII of Brand [1], which is a very good introduction to undergraduate vector and tensor analysis. The remainder consists of a few results from the more-specialized Weatherburn [2].

The current distribution is defined by a real vector field $J(x)$, nonzero for all $x = (x_1, x_2, x_3)$ in some bounded domain D, an open subset of euclidean space. The wire at x is required only to be parallel to $J(x)$, so we can replace J by the normalized vector field

$$T(x) = J(x)/\,|J(x)|.$$

A *layering* of T (and J) is defined by a real-valued function $L(x)$ such that ∇L is nonzero and perpendicular to $T(x)$ for all x in D. The layers will be the level-surfaces $L = c$ of L.

L and T are assumed to have continuous first derivatives but, like the nineteenth century geometers, we will not explicitly state all continuity conditions. Also, we will use their phrase "in general" to avoid analysis of singular situations; and we will use integrability only as a local concept.

The wires can be represented by solutions of the differential equation

$$\frac{dr(s)}{ds} = T(r(s)). \tag{1}$$

L is an integral of the equation. $L(r(s))$ is constant for all s (and a fixed solution curve r). s measures arc length along r. $T(x)$ is the unit tangent vector to r at $r(x)$.

In the solenoidal example above, with the helical wiring approximated by circles (degenerate helices), D can be taken as the annulus

$$0 < a < \sqrt{x_1^2+x_2^2} < b,$$

$$0 < x_3 < h$$

with

$$T(x) = (-x_2, x_1, 0) \,/\, \sqrt{x_1^2+x_2^2}.$$

The solutions of (1) will be circles with centers on the axis of the cylinder, so the function

$$L(x) = \sqrt{x_1^2+x_2^2}$$

will be a layering of T. The stress exerted on the underlying layer by the corresponding wire will be directed radially inwards.

For the tangent field T corresponding to an arbitrary current distribution, that force will be directed along the principal normal N to the curve, defined by

$$N(r(s)) = \frac{dT(r(s))}{ds} \,/\, \kappa(x)$$

where κ is a non-singular scalar called the *curvature* of r at $r(s)$. By the definition of integrability of a vector field, an L will exist everywhere perpendicular to this force if and only if N is integrable. By the classical theorem, this is true only if $N \cdot curl N = 0$. (The converse would involve non-local considerations that have been explicitly excluded from this paper.)

The component of $curl N$ in the N-direction at x is

$$((B(x)\cdot\nabla)N(x))\cdot T(x) - ((T(x)\cdot\nabla)N(x))\cdot B(x). \qquad (2)$$

But $N(x)\cdot T(x) = 0$ for all x in D, so

$$\nabla(N(x)\cdot T(x)) = (\nabla N(x))\cdot T(x) + N(x)\cdot(\nabla T(x)) = 0$$

and the first term of (2) is equal to

$$-((B(x)\cdot\nabla)T(x))\cdot N(x).$$

By the second Frenet formula, the second term in (2) is equal to

$$-(-\kappa(x)T(x)+\tau(x)B(x))\cdot B(x) = -\tau(x)B(x)\cdot B(x) = -\tau(x)$$

where $\tau(x)$ is the torsion at $x = r(s)$ of the curve r through x.

Therefore

$$((B(x)\cdot\nabla)T(x))\cdot N(x) + \tau(x) = 0 \tag{3}$$

is equivalent to $N\cdot curlN = 0$. However (3) is easier to use in computing the windability of T. (The expression on the left of (3) has an intuitive geometrical meaning. Associated with every curve is a strip called the rectifying strip. It contains the curve and also contains $B(x)$ in its tangent space. It is contained in the envelope of the s-parameterized family of planes, the rectifying planes, each of which contains $r(s)$ and, as tangents, $B(x)$ and $T(x)$. This strip is determined by r alone, independently of the family of curves in which it is imbedded. But that family also determines a strip containing r, made up of those curves in the family that are tangent to the rectifying strip "along" $B(r(s))$ "at" $r(s)$. The expression in (3) measures the rate of deviation of that strip from the rectifying strip at that point. When it is zero the two strips coincide. Then N is integrable and its integral surfaces can be constructed by putting together the rectifying strips.)

We first define a quantity $w(T,L)$ that measures the windability of T in a given layering L:

$$w(T,L) = \max_{x \text{ in } D} \cot \ |angle\,(N(x),\nabla L(x))|.$$

Then define the *windability* W by

$$W(T) = \min_L w(T,L). \tag{4}$$

The tangent of $angle\,(N(x),\nabla L(x))$ is the smallest coefficient of friction between layers that would keep the wire at x from slipping laterally. Therefore, $W(T)$ is the reciprocal of the smallest coefficient of friction needed to keep the wire from slipping in an optimal layering -- optimal in the sense that there exists no layering of T in which an even smaller coefficient of friction would hold the coil together during winding.

L defines an optimal layering if the maximum of the angle between its gradient and N is as small as possible. To construct such an L, we first construct each layer by putting together strips along each curve r (solution of (1)) in the layer. Each strip is oriented so that the maximum along r of the angle between its normal and the principal normal to r is minimal.

To carry out this construction, define two points in D to be equivalent with respect to the relation R if and only if they are both on the same curve r. Define D/R to be the two-dimensional manifold whose "points" are the curves r. In the terminology of [2], section 103, D/R is a curvilinear congruence. In modern terminology it is the quotient manifold of D by R. (There may be no cross-section of the family of curves that would allow us to identify points of D/R with points in a two-dimensional submanifold of D.) Any layering function L is constant on each r (in the terminology of [2], section 104, it is a "surface of the congruence"), so it is compatible with the relation R and defines a function L/R on D/R. There is a one-to-one correspondence between layers (surfaces) in D and curves in D/R. The set of all curves in D/R tangent at a given point r (layers in D tangent along a given curve r) determines a direction at r (strip along r). We want to vary that direction until the corresponding strip is optimal.

A convenient way to formalize the argument is provided by the notation of differential forms. Let g be a linear differential form on D/R. (In a subset of D/R where u_1 and u_2 can serve as coordinates, there will exist functions $f_1(u_1,u_2)$ and $f_2(u_1,u_2)$ such that the form can be written $g = f_1(u_1,u_2)du_1+f_2(u_1,u_2)du_2$.) Let Q be the quotient mapping from D to D/R. Then g defines a linear form $(dQ)^* g$ on D that can be written $G(x)\cdot dx$ for some vector $G(x)$ perpendicular to that curve r in the congruence such that $x = r(x)$. (No metric has been defined on D/R, so there is no invariant connection between forms g and vectors (f_1,f_2).) Because $N(x)$ and $B(x)$ span the space of vectors perpendicular to r at $r(s)$,

$$G(x) = e_1(x)N(x) + e_2(x)B(x)$$

for some scalar functions e_i. Define $\Theta_g(x) = angle(G(x),N(x))$, and then define g_0 as the form g such that at each r in D/R,

$$\max_g |\Theta_g(r(s))|$$

is minimal. But all linear differential forms on two-dimensional manifolds are integrable (locally). In general they are multiples, by an integrating factor, of the differential of a function. And every function on D/R is of the form L/R for some layering function L on D. Let L_0 be that function on D which corresponds to g_0. Then L_0 determines an optimal layering of T:

$$W(T) = w(T, L_0).$$

To illustrate this procedure, we apply it to the skewed solenoid. It is obtained from the solenoid by the affine transformation

$$(x_1, x_2, x_3) \to (x_1 + cx_3, x_2, x_3)$$

Each circle τ is transformed into another circle in the same plane, and

$$N(x) = (cx_3 - x_1, -x_2, 0) \,/\, \sqrt{(cx_3 - x_1)^2 + x_2^2}$$

is still the unit vector pointing towards the center of that (new) circle.

$$T(x) = (-x_2, x_1 - cx_3, 0) \,/\, \sqrt{(cx_3 - x_1)^2 + x_2^2}$$

is obtained by rotating it through a right angle. N is integrable if and only if (3) is satisfied. But $\tau = 0$ for any family of circles; and $B(x) = (0,0,1)$, and in applying the product rule for differentiation to T the term containing the derivative of the denominator with respect to $B(x) = d/dx_3$ can be ignored because the numerator is perpendicular to $N(x)$. Therefore, (3) becomes

$$\left\{\frac{d}{dx_3}(-x_2, x_1 - cx_3, 0)\right\} \cdot (cx_3 - x_1, -x_2, 0) \,/\, ((cx_3 - x_1)^2 + x_2^2) = 0$$

and

$$cx_2 \,/\, ((cx_3 - x_1)^2 + x_2^2) = 0.$$

Thus there is no way to wind a skewed (nonzero c) solenoid such that the wire is a geodesic in each layer.

The best that can be done in this case is a layering L_0 that minimizes the maximum over D of $angle(\nabla L, N)$. By the symmetry of T, we need look only at

$$\max_s angle((gradL)(r(s)), N(r(s)))$$

for any one solution r of (1). It can be taken as the circle in the $x = 0$ plane with radius a and center at (0,0,0), parameterized such that $r(0) = (a, 0, 0)$.

In this case there does exist a cross-section of the curvilinear congruence that can be identified with D/R, namely, the parallelogram in the $x_2 = 0$ plane defined by $0 \le x_3 \le h$ and $a + cx_3 \le x_1 \le b + cx_3$ (the intersection of D with the positive quadrant in the $x_2 = 0$ plane). Let $u_1 = x_1$ and $u_2 = x_3$. Define the value of the linear differential form g at $(a, 0)$ by $f_1(a,0) = \cos\alpha$ and $f_2(a,0) = \sin\alpha$. Then $\Theta_g(r(s))$ could be obtained by integrating along r the expression we have obtained for $N \cdot curlN$; but it is a monotonic function of s between 0 and πa, so its absolute value attains its maximum at one of these two points. That absolute value is minimal when $\alpha = -arctan\ c$, so

$$W(T) = w(T, L) = 1/c$$

and L_0 can be taken as the transform of $\sqrt{x_1^2 + x_2^2}$. The layers that were right circular cylinders are transformed into cylinders with elliptical cross-sections.

OPTIMIZATION OF THE WINDABILITY

The coefficient of friction may be too low to hold the coil together during winding even in an optimal layering. One would like to be able to increase the windability, necessarily by altering the distribution; without unduly distorting the magnetic field in the region of interest.

It is clear that a single isolated wire can always be supported in place: a curve r can always be imbedded in a narrow strip perpendicular at each $r(s)$ to $N(r(s))$. Then r is a geodesic in that surface. So the windability of an isolated current filament is always infinite, no matter how convoluted it may be. But this does not take care of current distributions defined on sets with non-empty interior. For them we want to put the strips together to form layers. As a first step in that direction, we would like the strip to support a layer of parallel wires, i.e., a one-parameter family of parallel geodesics. Fortunately, a great deal is known about such surfaces. By the theorem on page 25, volume II of [2],

"If a family of curves are both geodesics and parallels on a surface, the surface is developable."

A surface is *developable* if and only if it is the envelope of a one-parameter family of planes. Then it will have the useful properties that it is swept out by its only family of rulings and that it can be unrolled out flat onto a plane without distortion. Its intrinsic geometry is that of a plane. All developable surfaces are (extrinsically) either planes, cylinders, cones or else they are the surface swept out by the tangents to some twisted curve in space, its edge of regression.

Associated with every curve are several developables other than its tangent surface. One of them is its "rectifying developable", containing the rectifying strip described after equation (3). It is the envelope, as s varies, of the rectifying plane, orthogonal to $N(r(s))$ at $r(s)$. r is geodesic in its rectifying developable; hence it can be viewed as a straight line in a plane, the plane with which the rectifying developable is isometric, and can be imbedded in a one-parameter family of straight lines in that plane, i.e., of geodesics in its rectifying developable. Let $S_{r,0}$ be this strip of geodesics, of width 2ε with r the center-line.

In the next layer wound lengthwise on top of this strip, the wires will naturally tend to align themselves with the wires below, even though this may cause them to deviate sightly from geodesicity in their own layer. For a mathematical model of such a multi-layered strip, imbed $S_{r,0}$ in a one-parameter family $S_{r,\lambda}$, $-\varepsilon< \lambda <\varepsilon$ of parallel surfaces of width 2ε by the following construction: Using $S_{r,0}$ as the "director surface" ([2], page 183), construct the two-parameter family of straight lines (rectilinear congruence) normal to $S_{r,0}$ (a "normal system" in the terminology of [1], section 141). Define $L(x) = \lambda$ if and only if the signed distance of x from $S_{r,0}$ equals λ. This distance will necessarily be measured along the straight line of the congruence through x. It is assumed that ε is small enough to exclude singular points in the congruence. The level surfaces $L = \lambda$ define the one-parameter family of parallel surfaces. Those lines in the congruence that intersect a given geodesic in $S_{r,0}$, form a one-parameter subfamily that intersects $S_{r,\lambda}$ in a curve, thus setting up a one-to-one correspondence between the family of geodesics in $S_{r,0}$ and a family of curves in $S_{r,\lambda}$.

Define the domain D_r as the union of the sets $S_{r,\lambda}$ for $-\varepsilon < \lambda < \varepsilon$, and define T_r, N_r, and B_r in D_r as the tangent, normal, and binormal fields of the two-parameter family of curves. Let r_0 be one of the geodesics in the defining family of $S_{r,0}$. Then the corresponding curve in $S_{r,\lambda}$ is given by

$$r_\lambda(s) = r_0(s) + \lambda N_r(r_0(s)) \; .$$

because $N_r(r(s))$ is perpendicular to $S_{r,0}$ (by the geodesicity of r_0); and

$$\frac{dr_\lambda(s)}{ds} = T_r(r_0(s)) + \lambda \frac{dN_r(r_0(s))}{ds} .$$

But

$$\frac{dN_r(r_0(s))}{ds} = -\kappa_0(s)T_r(r_0(s)) + \tau_0(s)B_r(r_0(s))$$

by the second Frenet formula. Therefore

$$\frac{dr_\lambda(s)}{ds} = (1-\lambda\kappa_0(s))T_r(r_0(s)) + \lambda\tau_0(s)B_r(r_0(s)).$$

As the presence of the second term on the right hand side of this formula shows, r_λ need not be a geodesic in $S_{r,\lambda}$. That family need not even consist of parallel curves, though this latter is a higher order effect.

By normalizing and differentiating this expression for $dr_\lambda(s)/ds$, the principal normal $N_r(r_\lambda(s))$ at $r_\lambda(s)$ can be found, and then the angle between it and $N_r(r_0(s))$ which is also normal to $S_{r,\lambda}$ at $r_\lambda(s)$. Then $w(T_r, L_r)$ and $W(T_r)$ can be found. The angle is a continuous function of λ that goes to zero with ε, so by a compactness argument it can be shown that

$$\lim_{\varepsilon \to 0} W(T_r) = \infty. \tag{5}$$

The construction of $S_{r,\lambda}$ is greatly facilitated by the developability of $S_{r,0}$. Coordinatize the strip $S_{r,0}$ by developing it out flat in the plane as a rectangle and defining the s-axis parallel to the long dimension and at a distance ε from either long edge. Then take for the second set of coordinate lines the one-parameter family of straight lines crossing the s-axis at an angle whose tangent is $\kappa(s)/\tau(s)$, where $\kappa(s)$ and $\tau(s)$ are the curvature and torsion of the given curve r at $r(s)$. This is the angle at which the Darboux vector intersects r. Define the coordinates of a point in the flattened strip as s and s_1, where s_1 is the signed distance of the point from the s-axis as measured in the direction $arctan\ (\kappa(s)/\tau(s))$. If ε is small enough, the coordinates will be unique. Now define an isometry of the flattened strip into D_r by mapping the point with coordinates (s,s_1) in the strip into the point $r(s) + s_1\delta(s)/|\delta(s)|$ in D_r, where $\delta(s)$ is the Darboux vector. This is the mapping that identifies the straight lines parallel to the s-axis in the flattened strip with the defining family of geodesics in $S_{r,0}$. The Darboux vector at s lies along the ruling of $S_{r,0}$ at s.

The mapping can be extended, though it need no longer be an isometry off of $S_{r,0}$, to a mapping of a rectangular parallelepiped with square cross-section, 2ε on each side, and with the flat s,s_1-strip as its midplane, by mapping the point (s,s_1,λ) in the rectangular parallelepiped into the point $r(s) + s_1\delta(s)/|\delta(s)| + \lambda N(s)$ in D. Then the straight lines parallel to the s-axis in the rectangular parallelepiped correspond to the bundle of wires in D_r. The rulings in $S_{r,\lambda}$ consist of those points closest to the rulings in $S_{r,0}$, because the enveloping planes of parallel developable surfaces correspond and are constant along a ruling.

In many schemes for the numerical computation of magnetic fields, the continuous current distribution J is, in effect, approximated by a finite number of one-dimensional, discrete current elements. For example, if the relationship between current distribution and magnetic field is linear and implemented by an integral transform, the Riemann sum approximating to the integral can be taken over such current elements, suitably weighted.

Although it may or may not be numerically convenient, the current elements can be joined together to form a finite number of current filaments along the curves $r_1, r_2, \cdots, r_N$, approximating J to any desired degree of accuracy in the sense of numerical convergence of the magnetic field calculation.

The current filament along the curve r_i can be approximated to an arbitrary degree of accuracy as far as the magnetic field in the region of interest is concerned (necessarily in a region bounded away from that of the coil) by a continuous current distribution J_i in some domain D_{r_i}, union of the sets $S_{r_i,\lambda}$ over $-\varepsilon < \lambda < \varepsilon$, as described above. Further, if all the ε are small enough, $W(J_i)$ can be made larger than any preassigned constant by (5), and hence so can $W(\Sigma_i J_i) = \min_i W(J_i)$. By making these remarks mathematically rigorous, one can prove the main theorem of this paper:

Every current distribution can be approximated arbitrarily closely by a current distribution of arbitrarily high windability.

To carry out such a proof one could define linear spaces C of current distributions and M of magnetic fields, and a weak topology on M induced by test functions with support in the region where the magnetic field is of interest. Define a topology on C as the weakest in which the transformation from C to M is continuous, and show that the set of all J in C such that $W(J) > c$ is dense in C for any constant c. Modern functional analysis provides us with several kinds of locally convex linear topological spaces for which such a proof could be devised, but it would not convey any useful insights. The heart of the proof is contained in the intuitive argument given above.

In fact, the construction of the approximating distributions $\Sigma_i J_i$ is essentially just that already used on the shop floor of magnet-winding facilities. Even difficult coil configurations are put together from a relatively small number of domains D_{r_i}, each of which is wound in layers of parallel wires built up until deviation from geodesicity endangers its cohesiveness. The cross-sections of the D_{r_i} are usually trapezoidal or wedge-shaped (rather than square) so as to fit roughly together.

COMPUTER AIDED DESIGN AND MANUFACTURE

The real purpose of this paper is to argue that the whole design and manufacture process for magnetic coils, from the magnetic field calculation to the running of the numerically controlled milling machine that shapes the supports, can be aided by a unified system of computer programs. Both ends of this process have already been computerized. Programs are available and heavily relied upon for field calculations; and the milling machine is, of course, controlled by a program. Classical differential geometry can be used to help bridge part of the gap between these two ends by putting a mathematical foundation under the art of magnet winding. The present cut-and-try procedure of experimenting with epoxy or aluminum models before committing to the more expensive steel supports, can be replaced by experimentation with mathematical models. Experience and intuition cannot be replaced, but software is a cheaper and more flexible material for them to work on than hardware.

REFERENCES

[1] Louis Brand, *Vector and Tensor Analysis*, John Wiley & Sons, Inc., New York (1947).

[2] C. E. Weatherburn, *Differential Geometry of Three Dimensions*, Cambridge University Press, Cambridge (1955).